Estimation of Siderostat Gains Using NAT and Fringe Data

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1.0 Introduction

In this brief document, I estimate the siderostat gains versus siderostat and channel. I create expressions relating the zero-spacing fringe counts versus the NAT counts and solve for the gains via least squares.

Assumption: There are a sufficient number of calibrator bias scans for a non-degenerate solution.

Assumption: Photon number variability versus scan (resulting from atmospheric effects, pointing, and the star flux due to ~ blackbody temperature) is sufficient for a non-degenerate solution.

Assumption: The NAT counts photometry is of sufficient accuracy (no lost counts).

2.0 Initial Mathematics

Assumption: The effective area of the telescope is independent of wavelength.

Assumption: The angular area of the star is ~ independent of wavelength.

The number of photons at the top of the atmosphere is given by

$$N(\lambda)\,d\lambda = \Delta t\,A\,\Omega\,n(\lambda)\,d\lambda\,,$$

where λ is the wavelength (μm), Δt is the integration time (s, NAT or BC superscript), A is the effective area of the telescope (m^2), $d\lambda$ is the wavelength differential interval (μm), Ω is the angular area of the calibrator star (ster), and $n(\lambda)$ is the photon number density (photon s^{-1} m^{-2} μm^{-1} $ster^{-1}$). In general, the photon number density can depend on parameters in addition to the wavelength.

Assumption: The calibrator stars are approximately blackbody radiators.

For a blackbody radiator, the photon number density becomes

$$n(\lambda) \rightarrow n(\lambda;T) = \frac{2c}{\lambda^4}\frac{1}{e^{hc/\lambda kT} - 1}\,,$$

where c is the speed of light (299792458 m s^{-1}, exactly), h is the Planck constant (6.62606957 x 10^{-34} J s), k is the Boltzmann constant (1.3806488 x 10^{-23} J K^{-1}), and T is the temperature (K). The units clearly contain length^{-3}, but they should be converted to m^{-2} μm^{-1}.

The number of photons at a detector for siderostat *m* and scan *n* (NAT or BC superscript) may be expressed as

$$N_{mn}(\lambda)\,d\lambda = \left[G\,\beta_n(\lambda)\,\gamma_n(\lambda)\,\delta_{mn}(\lambda)\,\varepsilon_m(\lambda)\,\zeta_m(\lambda)\,\eta(\lambda)\right]N_n(\lambda)\,d\lambda = \mu_{mn}(\lambda)\,N_n(\lambda)\,d\lambda\,,$$

where G is the conversion factor between counts and photons (NAT or BC superscript), $\beta_n(\lambda)$ is the atmospheric extinction gain (how much "stuff" is in the atmosphere), $\gamma_n(\lambda)$ is the sec(z)-dependent gain, $\delta_{mn}(\lambda)$ is the pointing gain due only to the siderostat mirror, $\varepsilon_m(\lambda)$ is the common-optics gain (due to the optics from the NAT mirror to the point where the light is split between the NAT and beam combiner), $\zeta_m(\lambda)$ is the gain of either the NAT or beam combiner feed optics (NAT or BC superscript), $\eta(\lambda)$ is the detector gain (NAT or BC superscript), $\mu_{mn}(\lambda)$ is the product of all gains (NAT or BC superscript), and $N_n(\lambda) \rightarrow N_n(\lambda;T_n) = \Delta t\,A\,d\lambda\,\Omega_n\,n(\lambda;T_n)$. This expression must be integrated over wavelength for both the NAT and BC subsystems. For the sake of convenience, I will define wavelength-averaged gain products.

The wavelength-integrated number of photons detected at each NAT is

$$\overline{N}_{mn}^{NAT} = \int_{\Delta\lambda_{NAT}} d\lambda\,N_{mn}^{NAT}(\lambda) = \int_{\Delta\lambda_{NAT}} d\lambda\,\mu_{mn}^{NAT}(\lambda)\,N_n(\lambda)\,,$$

where $\Delta\lambda_{NAT}$ is the approximate width of the NAT bandpass. I define the wavelength-averaged gain product as

$$\overline{\mu}_{mn}^{NAT} = \frac{\int_{\Delta\lambda_{NAT}} d\lambda\,\mu_{mn}^{NAT}(\lambda)\,N_n(\lambda)}{\int_{\Delta\lambda_{NAT}} d\lambda\,N_n(\lambda)} = \frac{\overline{N}_{mn}^{NAT}}{\overline{N}_n}\,,$$

which means that

$$\overline{N}_{mn}^{NAT} = \overline{\mu}_{mn}^{NAT}\,\overline{N}_n\,.$$

This quantity IS measured directly by each NAT.

The mathematics for the zero-spacing wavelength-integrated number of photons from the BC are identical. The wavelength-integrated number of photons detected at each BC is

$$\overline{N}_{mnp}^{BC} = \int_{\Delta\lambda_p} d\lambda\,N_{mn}^{BC}(\lambda) = \int_{\Delta\lambda_p} d\lambda\,\mu_{mn}^{BC}(\lambda)\,N_n(\lambda)\,,$$

where $\Delta\lambda_p$ is the approximate width of the BC bandpass centered about the wavelength of the p^{th} channel λ_p. I define the wavelength-averaged gain product as

$$\overline{\mu}_{mnp}^{BC} = \frac{\int_{\Delta\lambda_p} d\lambda\,\mu_{mn}^{BC}(\lambda)\,N_n(\lambda)}{\int_{\Delta\lambda_p} d\lambda\,N_n(\lambda)} = \frac{\overline{N}_{mnp}^{BC}}{\overline{N}_n}\,,$$

which means that

$$\overline{N}_{mnp}^{BC} = \overline{\mu}_{mnp}^{BC}\,\overline{N}_n\,.$$

This quantity IS NOT measured directly by each BC. There sum over siderostats IS measured by each BC

$$\overline{N}_{np}^{BC} = \sum_{m=1}^{4} \overline{\mu}_{mnp}^{BC} \overline{N}_n = \overline{\mu}_{np}^{BC} \overline{N}_n .$$

In the next section, we equation the NAT and BC observables and set up a least-squares problem.

3.0 Least-Squares

Consider the ratio of the numbers of photons from the previous section

$$\frac{\overline{N}_{mnp}^{BC}}{\overline{N}_{mn}^{NAT}} = \frac{\overline{\mu}_{mnp}^{BC} \overline{N}_n}{\overline{\mu}_{mn}^{NAT} \overline{N}_n} = \frac{\overline{\mu}_{mnp}^{BC}}{\overline{\mu}_{mn}^{NAT}},$$

which can be rearranged as

$$\overline{N}_{mnp}^{BC} = \frac{\overline{\mu}_{mnp}^{BC}}{\overline{\mu}_{mn}^{NAT}} \overline{N}_{mn}^{NAT} = \xi_{mnp} \overline{N}_{mn}^{NAT} .$$

When this equation is summed over siderostat, I obtain

$$\overline{N}_{np}^{BC} = \sum_{m=1}^{4} \overline{N}_{mnp}^{BC} = \sum_{m=1}^{4} \xi_{mnp} \overline{N}_{mn}^{NAT} .$$

Both $\overline{N}_{np}^{BC}$ and $\overline{N}_{mn}^{NAT}$ are measured quantities stored in *.cha files (actually, they are stored as point data, so they must be scan averaged first), and the ξ_{mnp} are unknowns. Even if I set up a separate system of equations for each *p*, there are too many unknowns to perform a least-squares solution for each *p*. I will study the ξ_{mnp} in detail to see if there are any simplifications.

Assumption: The NAT wavelength-averaged gain products can be approximately written in terms of wavelength-averaged factors, $\overline{\mu}_{mn}^{NAT} \approx G^{NAT} \overline{\beta}_n \overline{\gamma}_n \overline{\delta}_{mn} \overline{\varepsilon}_m \overline{\zeta}_m^{NAT} \overline{\eta}^{NAT}$.

Assumption: The BC wavelength-averaged gain products can be approximately written in terms of wavelength-averaged factors, $\overline{\mu}_{mnp}^{BC} \approx G^{BC} \overline{\beta}_{np} \overline{\gamma}_{np} \overline{\delta}_{mnp} \overline{\varepsilon}_{mp} \overline{\zeta}_{mp}^{BC} \overline{\eta}_p^{BC}$.

Given the previous two assumptions, the unknowns can be written as

$$\xi_{mnp} \approx \frac{G^{BC}}{G^{NAT}} \frac{\overline{\beta}_{np}}{\overline{\beta}_n} \frac{\overline{\gamma}_{np}}{\overline{\gamma}_n} \frac{\overline{\delta}_{mnp}}{\overline{\delta}_{mn}} \frac{\overline{\varepsilon}_{mp}}{\overline{\varepsilon}_m} \frac{\overline{\zeta}_{mp}^{BC}}{\overline{\zeta}_m^{BC}} \frac{\overline{\eta}_p^{BC}}{\overline{\eta}^{NAT}} .$$

The β, γ, and δ ratios are the only ones that depend on the scan number *n*.

Assumption: The variability of the $\overline{\beta}_{np}$ and $\overline{\beta}_n$ versus scan are approximately the same, so that their ratio is independent of scan.

Assumption: The variability of the $\overline{\gamma}_{np}$ and $\overline{\gamma}_n$ versus scan are approximately the same, so that their ratio is independent of scan.

Assumption: The variability of the $\overline{\delta}_{mnp}$ and $\overline{\delta}_{mn}$ versus scan are approximately the same, so that their ratio is independent of scan.

Using these assumptions, I find that the unknowns are independent of scan, or $\xi_{mnp} \approx \xi_{mp}$, which means that the equations relating the photon numbers are

$$\overline{N}_{np}^{BC} = \sum_{m=1}^{4} \xi_{mp} \, \overline{N}_{mn}^{NAT} \ .$$

For each *p*, this can be set up as a matrix equation

$$\begin{bmatrix} \vdots \\ \overline{N}_{np}^{BC} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ \overline{N}_{1n}^{NAT} & \overline{N}_{2n}^{NAT} & \overline{N}_{3n}^{NAT} & \overline{N}_{4n}^{NAT} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \bullet \begin{bmatrix} \xi_{1p} \\ \xi_{2p} \\ \xi_{3p} \\ \xi_{4p} \end{bmatrix}$$

or

$$\vec{N}_{p}^{BC} = \overleftrightarrow{N}^{NAT} \bullet \vec{\xi}_{p} \ .$$

The length of the vector on the left hand side is the number of scans. The shape of the matrix is the number of scans by number of siderostats (4). The length of the vector on the right hand size is the number of siderostats (4). Solving this system of equations is equivalent to least squares. It can be weighted according to the errors of the BC number of photons for each row, if desired.

One the ξ_{mp} are determined, it is possible to estimate the BC number of photons for each siderostat, scan, and channel

$$\overline{N}_{mnp}^{BC} \approx \xi_{mp} \, \overline{N}_{mn}^{NAT} \ .$$

These numbers are used for the visibility bias correction defined in Hummel *et al.* (2003).